

Math 53: Worksheet 3

October 9

1. Compute:
 - (a) $D_{\mathbf{u}}f(1, 1)$ where $f(x, y) = \frac{x}{y}$ and $\mathbf{u} = (\frac{3}{5}, \frac{-2}{5})$.
 - (b) ∇f for $f(x, y, z) = x^2 \sin(yz)$.
 - (c) $D_{\mathbf{u}}g(4, -1, \pi)$ where $g(p, q, r) = p^2 \sin(r) - q \cos(r)$ and $\mathbf{u} = (1, 5, -4)$.
 - (d) Equation of tangent plane to the surface $xy + yz + zx + 3 = e^{xyz}$ at $(-1, 2, 0)$.
2. Let $\mathbf{u} = (a, b)$ be a unit vector and let $f(x, y)$ have continuous second-order partial derivatives. Find an expression for $D_{\mathbf{u}}(D_{\mathbf{u}}f(x, y))$.
3. Prove the (very obvious) fact that every line normal to the sphere $x^2 + y^2 + z^2 = r^2$ passes through the origin. Can you show that any surface for which every line normal passes through the origin must be a sphere of this form?
4. The temperature T in an infinite ball is inversely proportional to the distance from the center of the ball, which we take to be the origin. The temperature at the point $(2, 1, 2)$ is 60° F.
 - (a) Find the rate of change of T at $(-2, 1, 0)$ in the direction toward the point $(4, 1, -1)$.
 - (b) Show that at any point in the ball the direction of greatest increase in temperature is given by a vector that points toward the origin.
5. Consider a swimming pool with the temperature of the water at (x, y, z) given by $F(x, y, z)$. A fish swims through the water with position at time t given by $p(t)$.
 - (a) The fish feels a temperature at any point in time. How fast does the temperature that the fish feels change, at time 0?
 - (b) What is the directional derivative of F in the direction that the fish is traveling in at time 0? What is the speed (NOT velocity) of the fish at time 0?
 - (c) At $t = 1$, the fish decides it is happy with its current temperature. Describe/specify a set of directions (vectors) in which the fish should swim.
 - (d) The fish changes its mind instantaneously at time $t = 1$. It goes in the direction such that the water gets colder, fastest. Give a vector pointing in this direction.
 - (e)* Let $F(x, y, z) = x^2 + 2y^2 + 3z^2$. The fish continues swimming (see (d)) in the direction of coldest water (at each time). Parameterize the fish's path (for $t \geq 1$).